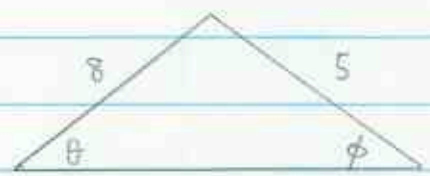
16. Derive formula for $\sin(\theta - \frac{\pi}{2})$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$
$$\sin(\theta - \tfrac{\pi}{2}) = \sin(\theta)\cos(\tfrac{\pi}{2}) - \cos\theta \sin(\tfrac{\pi}{2})$$
$$= \sin\theta[0] - \cos\theta[1]$$
$$\Rightarrow \quad \underline{\underline{\sin(\theta - \tfrac{\pi}{2}) = -\cos\theta}}$$

17. R5 p. 281



a) LAW OF COSINES, $\theta = 22^\circ$

$$5^2 = 8^2 + x^2 - 2(8)(x)\cos 22^\circ$$
$$0 = x^2 - (16\cos 22^\circ)x + 39$$
$$x = 11.4 \quad \text{or} \quad x = 3.4$$

b) $\sin\phi = 8\sin 85^\circ / 5 \approx 1.6$

$\sin\phi$ can't be $\approx 1.6 > 1$

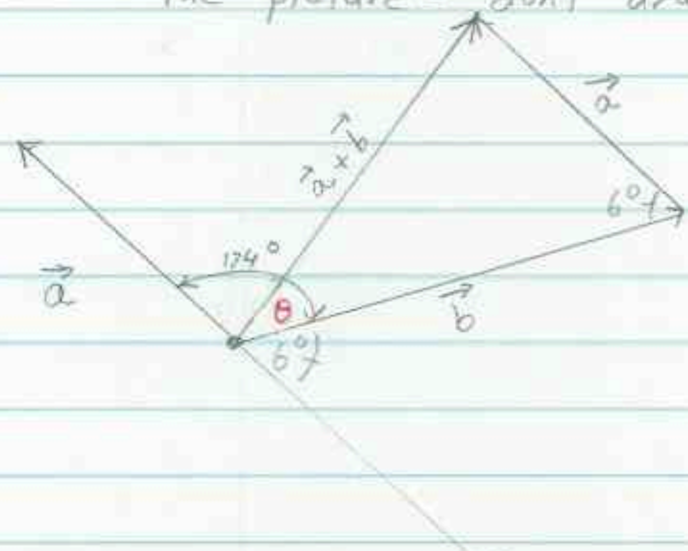
c) 5-cm side must be $\perp$ to 3rd side, w/ 8-cm side the hypotenuse of triangle.

$$\theta = \sin^{-1}(\tfrac{5}{8}) \approx 38.7^\circ$$

d) $x = 10.5$ cm, same as part (a) except $\theta = 47^\circ$ instead of 22°

17. R6

(a) PART A much easier if you exaggerate the picture - don't draw to scale...



PUT VECTORS HEAD TO TAIL

LAW OF COSINES: $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(6°)$

$$|\vec{a}+\vec{b}|^2 = 6^2 + 10^2 - 2(6)(10)\cos 6°$$

$$|\vec{a}+\vec{b}|^2 = 16.657$$

$$|\vec{a}+\vec{b}| = 4.08$$

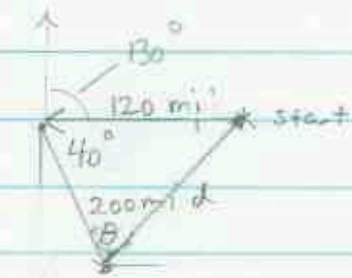
ANGLE θ: $\dfrac{\sin\theta}{|\vec{a}|} = \dfrac{\sin 6°}{|\vec{a}+\vec{b}|} \Rightarrow \theta = 8.842°$

ANGLE Between $\vec{a}$ and $\vec{a}+\vec{b}$ when tail to tail is $174° - 8.842° = \underline{\underline{165.2°}}$

17. R6 (b)

$$\vec{r} = \vec{a} + \vec{b} = (5+7)\vec{i} + (3-6)\vec{j}$$
$$\vec{r} = 12\vec{i} - 3\vec{j}$$
$$|\vec{r}| = \sqrt{144+9} = \sqrt{153} \approx 12.4$$
$$\theta = \tan^{-1}\left(\frac{-3}{12}\right) = -14.04^\circ$$ which is in 4^{th} quadrant.

(c)



$$d^2 = (200)^2 + (120)^2 - 2(200)(120)\cos 40^\circ$$
$$d^2 = 17629.87$$
$$d = 132.8 \text{ miles}$$ or use components

bearing of 165.5°

d) $d = 290.3$ km/h
bearing of 208.5°

I'll go easy on these type of problems – 1 at most – you can always use components...

Sorry there were so many of these....

18. show $\csc x = \cot x\,(\cos x + \tan x \sin x)$

$\cot x\,(\cos x + \tan x \sin x)$

$= \cot x \cos x + \cot x \tan x \sin x$

$= \dfrac{\cos x}{\sin x} \cos x + \sin x$

$= \dfrac{\cos^2 x}{\sin x} + \dfrac{\sin^2 x}{\sin x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\sin x}$

$= \dfrac{1}{\sin x}$

$= \csc x$

19. Show $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$

$$= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$

$$= \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x\,(1 - \cos^2 x)}{\cos^2 x}$$

$$= \sin^2 x \,\frac{(\sin^2 x)}{\cos^2 x}$$

$$= \sin^2 x \tan^2 x$$

20. Show $(\sin x + \cos x)^2 - 2 \sin x \cos x = 1$

$$(\sin x + \cos x)^2 - 2 \sin x \cos x$$

$$= \sin^2 x + \cancel{2 \sin x \cos x} + \cos^2 x - \cancel{2 \sin x \cos x}$$

$$= \sin^2 x + \cos^2 x$$

$$= 1$$