Accelerated Math 3

Topics and Review - Test 3

1. $f(x) = \dfrac{3x^2 - x}{2x^2 - 3x + 1}$

a) vertical asymptotes: $2x^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$

$x = \frac{1}{2}$ or $x = 1$

b) horizontal asymptotes $y = \dfrac{3x^2}{2x^2} = \dfrac{3}{2}$

$\boxed{y = \frac{3}{2}}$

c) x-intercepts: $3x^2 - x = 0$

$x(3x-1) = 0$

$x = 0$ or $x = \frac{1}{3}$

$(0,0)$; $(\frac{1}{3}, 0)$

d) y-intercepts: $f(0) = \dfrac{3(0)^2 - 0}{2(0)^2 - 3(0) + 1} = \dfrac{0}{1} = 0$

$(0,0)$

e) end behavior: (same as horizontal asymptote)

as $x \to -\infty$ $f(x) = \frac{3}{2}$

as $x \to +\infty$ $f(x) = \frac{3}{2}$

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2. $f(x) = 4x-7$ $\quad g(x) = x^2-3x+2$

a) $g(f(3)) = g(4(3)-7) = g(5) = (5)^2-3(5)+2 = 25-15+2$

$= \underline{12}$

b) $f(g(3)) = f((3)^2-3(3)+2) = f(2) = 4(2)-7 = \underline{1}$

c) $f(g(x)) = 4(x^2-3x+2)-7$

$= 4x^2-12x+8-7$

$= \underline{4x^2-12x+1}$

d) $g(f(x)) = g(4x-7)$

$= (4x-7)^2-3(4x-7)+2$

$= 16x^2-56x+49-12x+21+2$

$= \underline{16x^2-68x+72}$

3. a) 137° $\quad$ ref $\angle = 180-137 = 43^\circ$

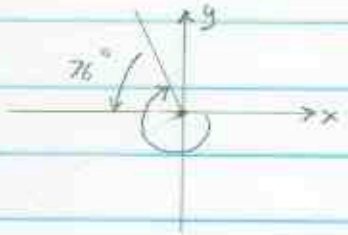
co-terminal: $137^\circ-360^\circ = -193^\circ$

$137^\circ+360^\circ = 497^\circ$

b) -256 $\quad$ ref $\angle = 76^\circ$

co-terminal: $-256+360 = 104^\circ$

$-256-360 = 616^\circ$



c) $\frac{5\pi}{12}$ $\quad$ ref $\angle = \frac{5\pi}{12}$

co-terminal: $\frac{5\pi}{12}+2\pi = \frac{29\pi}{12}$

$\frac{5\pi}{12}-2\pi = \frac{-19\pi}{12}$

d) $\frac{-7\pi}{2}$ $\quad$ ref $\angle = \frac{\pi}{2}$

co-terminal: $-\frac{7\pi}{2}+2\pi = \frac{\pi}{2}$

$-\frac{7\pi}{2}+-2\pi = -\frac{11\pi}{2}$

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4. No problems memorize! on review

5. a) $\sin^3 x + \sin x \cos^2 x = \sin x$

$= \sin x (\sin^2 x + \cos^2 x)$

$= \sin x (1)$

$= \sin x$ ✓

b) $\dfrac{1 + \tan^2 x}{\csc^2 x} = \tan^2 x$

$= \dfrac{1 + \tan^2 x}{\csc^2 x}$ pythagorean identity

$= \dfrac{\sec^2 x}{\csc^2 x}$

$= \tan^2 x$ ✓ quotient identity

c) $\dfrac{[(\sec x - 1)(\sec x + 1)]}{\sin^2 x} = \sec^2 x$

$= \dfrac{(\sec x - 1)(\sec x + 1)}{\sin^2 x}$

$= \dfrac{\sec^2 x - 1}{\sin^2 x}$

$= \dfrac{\tan^2 x}{\sin^2 x} = \tan^2 x \csc^2 x$

$= \dfrac{\sec^2 x}{\csc^2 x} \csc^2 x$

$= \sec^2 x$ ✓

Topics and Review - Test 3

5-(d) $(\sec^2 x + \csc^2 x) - (\tan^2 x + \cot^2 x) = 2$

$= 1 + \tan^2 x + 1 + \cot^2 x - \tan^2 x - \cot^2 x$

$= 1 + 1$

$= 2$ ✓

5(e) $\dfrac{1}{\sec x - 1} + \dfrac{1}{\sec x + 1} = 2\cot x \csc x$

$= \dfrac{\sec x + 1 + \sec x - 1}{\sec^2 x - 1}$

$= \dfrac{2\sec x}{\tan^2 x}$

$= \dfrac{2\sec x}{\frac{\sin^2 x}{\cos^2 x}} = 2\sec x \dfrac{\cos^2 x}{\sin^2 x}$

$= 2\dfrac{1}{\cos x}\dfrac{\cos^2 x}{\sin^2 x}$

$= 2\dfrac{\cos x \cdot 1}{\sin x \sin x}$

$= 2\cot x \csc x$ ✓

5(f) $\dfrac{\cos t}{1-\sin t} \geq \dfrac{1+\sin t}{\cos t}$

$$\frac{\cos t\,(1+\sin t)}{(1-\sin t)(1+\sin t)}$$

$$\geq \frac{\cos t\,(1+\sin t)}{1-\sin^2 t}$$

$$\geq \frac{\cos t\,(1+\sin t)}{\cos^2 t}$$

$$\geq \frac{(1+\sin t)}{\cos t} \quad \checkmark$$

5(g) $\sin^2 x \cos^3 x \geq (\sin^2 x - \sin^4 x)\cos x$

$$= (\sin^2 x - \sin^4 x)\cos x$$
$$= (1-\sin^2 x)\sin^2 x \cos x$$
$$\geq (\cos^2 x)\sin^2 x \cos x$$
$$\geq \sin^2 x \cos^3 x \quad \checkmark$$

5(h) $\dfrac{\cot v - 1}{\cot v + 1} \geq \dfrac{1-\tan v}{1+\tan v}$ note: $\cot v = \dfrac{1}{\tan v}$

$$\frac{\frac{1}{\tan v} - 1}{\frac{1}{\tan v} + 1}$$ (common denominator of $\tan v$)

$$= \frac{\frac{1-\tan v}{\tan v}}{\frac{1+\tan v}{\tan v}}$$ ✓ fraction division

$$\geq \frac{1-\tan v}{1+\tan v}$$

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Topics and Suggested Review - Test 3

6/

5(i) $\sin^4\theta - \cos^4\theta = \sin^2\theta - \cos^2\theta$

$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$

$= (\sin^2\theta - \cos^2\theta)(1)$

$= \sin^2\theta - \cos^2\theta$ ✓

#6 to come...

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Topics and Review - Test 3

7. a) $\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right) = -\frac{\pi}{6}$

b) $\tan^{-1}(\tan(\pi/3)) = \pi/3$ note $-\frac{\pi}{2} < \frac{\pi}{3} \leq \frac{\pi}{2}$

c) $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

d) $\sin^{-1}(\sin(\pi)) \neq \pi$ $\because -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

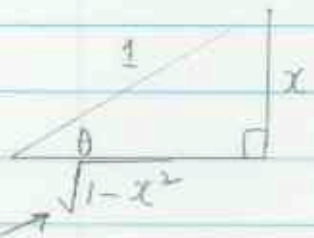
$\therefore \sin^{-1}(0) = 0$

e) $\cos(\tan^{-1}\sqrt{3})$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

f) $\cos^{-1}\left(\cos\frac{7\pi}{4}\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$ $\because$ range of $\cos^{-1}$ is $(0, \pi)$

8. a)



Draw unit circle:

$\tan\theta = \frac{opp}{adj} = \frac{x}{?}$

use Pythagorean theorem

$\cos\theta = \frac{adj}{hyp} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

b) $\sin(\tan^{-1}(x))$ same triangle

$\sin(\theta) = \frac{opp}{hyp} = \frac{x}{1} = x$

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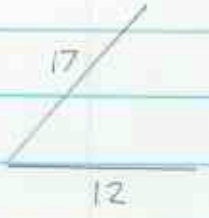
Topics and Test Review - Test 3

9.

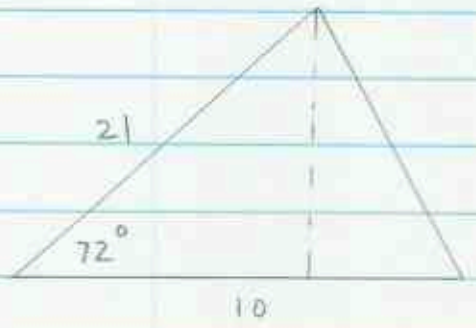


if you knew one of the angles you could use Law of Sines.

need at least one angle measure to make problem solvable.

my fault!

10.



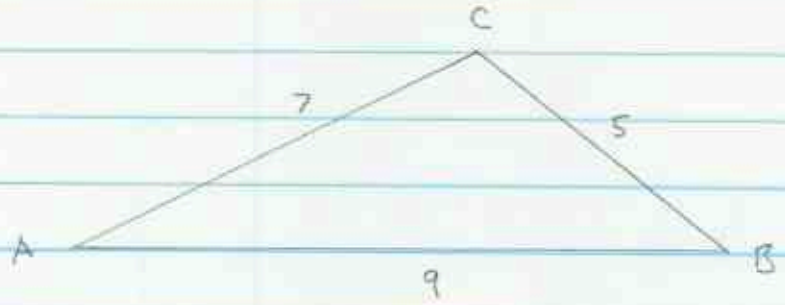
$$R = \frac{1}{2}(10)(21)\sin 72^\circ$$

$$= 5(21)\sin 72^\circ$$

$$= 105 \sin 72^\circ$$

$R \approx 99.86$ units^2

11.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$5^2 = 7^2 + 9^2 - 2(7)(9)\cos A$$

$$25 = 130 - 126\cos A$$

$$\frac{-105}{-126} = \cos A \qquad A = \cos^{-1}\left(\frac{105}{126}\right)$$

$A = 33.6^\circ$

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Topics and Test Review - Test 3

11. continued

$b^2 = a^2 + c^2 - 2ac\cos B$

$7^2 = 5^2 + 9^2 - 2(5)(9)\cos B$

$49 = 106 - 90\cos B$

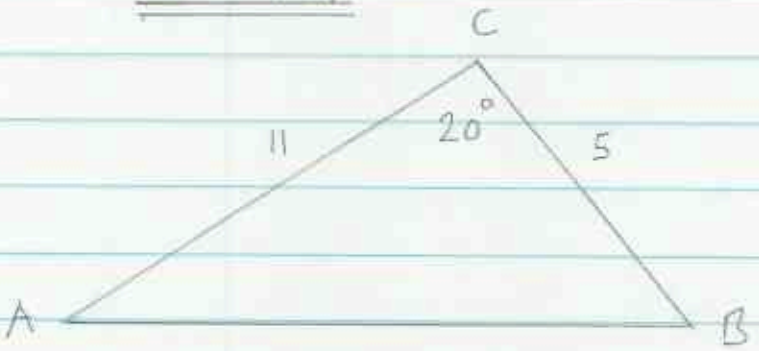
$\frac{57}{90} = \cos B \Rightarrow B = \cos^{-1}\left(\frac{57}{90}\right) = 50.7^\circ$

$B = 50.7^\circ$

$C = 180 - (A+B) = 180 - (33.6 + 50.7)$

$C \approx 95.7^\circ$

12.



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 25 + 121 - 2(5)(11)\cos 20^\circ$

$c^2 = 25 + 121 - 110\cos 20^\circ$

$c^2 = 42.634$

$c = 6.529$

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Topics and Test Review - Test 3

12. continued Find angle A

$a^2 = b^2 + c^2 - 2bc \cos A$

$25 = 121 + 42.634 - 2(11)(6.529) \cos A$

$0.9652 = \cos A$

$A = \cos^{-1}(0.9652)$

$A \approx 15.16^\circ$

$\therefore B = 180^\circ - (A + C) = 180 - (15.16 + 20)$

$= 144.8^\circ$